ELECTRO MAGNETIC THEORY
CHAPTER-1

Sources and effects of electromagnetic fields:

Electromagnetics (EM) is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied.

EM principles find applications in various allied disciplines such as microwaves, antennas, electric machines, satellite communications, bioelectromagnetics, plasmas, nuclear research, fiber optics, electromagnetic interference and compatibility, electromechanical energy conversion, radar meteorology and remote sensing. EM devices include transformers, electric relays, radio/TV, telephone, electric motors, transmission lines, waveguides, antennas, optical fibers, radars, and lasers.

1.1.Sources of electromagnetic fields

- **Natural sources Electromagnetic fields** are present everywhere in our environment but are invisible to the human eye. Electric fields are produced by the local build-up of electric charges in the atmosphere associated with thunderstorms. The earth's magnetic field causes a compass needle to orient in a North-South direction and is used by birds and fish for navigation.
- **Human-made sources**
  Besides natural sources the electromagnetic spectrum also includes fields generated by human-made sources. X-rays are employed to diagnose a broken limb after a sport accident. The electricity that comes out of every power socket has associated low frequency electromagnetic fields. And various kinds of higher frequency radio waves are used to transmit information – whether via TV antennas, radio stations or mobile phone base stations are some man-made sources.

- Background electromagnetic field levels from electricity transmission and distribution facilities. Electricity is transmitted over long distances via high voltage power lines. Transformers reduce these high voltages for local distribution to homes and businesses. Electricity transmission and distribution facilities and residential wiring and appliances account for the background level of power frequency electric and magnetic fields in the home.
- In homes not located near power lines this background field may be up to about 0.2 μT. directly beneath power lines the fields are much stronger. Magnetic flux densities at ground level can range upto several μT. Electric field levels underneath power lines can be as high as 10 kV/m. However, the fields (both electric and magnetic) drop off with distance from the lines. At 50m to 100 m distance the fields are normally at levels that are found in areas away from high voltage power lines. In addition, house walls substantially reduce the electric field levels from those found at similar locations outside the house.
- Electric appliances in the household, the strongest power frequency electric fields that are ordinarily encountered in the environment exist beneath high voltage transmission lines. In contrast, the strongest magnetic fields at power frequency are normally found very close to motors and other electrical appliances, as well as in specialized equipment such as magnetic resonance scanners used for medical imaging.

1.2: Health effects of electromagnetic fields

Exposure to electromagnetic fields is not a new phenomenon. However, during the 20thcentury, environmental exposure to man-made electromagnetic fields has been steadily increasing as growing electricity demand, ever advancing technologies and changes in social behaviour have created more and more artificial sources. Everyone is exposed to a complex mix of weak electric and magnetic fields, both at home and at work, from the generation and transmission of electricity, domestic appliances and industrial equipment, to telecommunications and broadcasting. Tiny electrical currents exist in the human body due to the chemical reactions that occur as part of the normal bodily functions, even in the absence of external electric fields. For example, nerves relay signals by transmitting electric impulses. Most biochemical reactions from digestion to brain activities go along with the rearrangement of charged particles. Even the heart is electrically active - an activity that your doctor can trace with the help of an electrocardiogram.

Low-frequency electric fields

This influence the human body just as they influence any other material made up of charged particles. When electric fields act on conductive materials, they influence the distribution of electric charges at their surface. They cause current to flow through the body to the ground.

Low-frequency magnetic fields

This induce circulating currents within the human body. The strength of these currents depends on the intensity of the outside magnetic field. If sufficiently large, these currents could cause stimulation of nerves and muscles or affect other biological processes.

Both electric and magnetic fields induce voltages and currents in the body but even directly beneath a high voltage transmission line, the induced currents are very small compared to thresholds for producing shock and other electrical effects.

Heating is the main biological effect of the electromagnetic fields of radiofrequency fields. In microwave ovens this fact is employed to warm up food. The levels of radiofrequency fields to which people are normally exposed are very much lower than those needed to produce significant heating. The heating effect of radio waves forms the underlying basis for current guidelines. Scientists are also investigating the possibility that effects below the threshold level for body heating occur as a result of long-term exposure. Till date, no adverse health effects from low level, long-term exposure to radiofrequency or power frequency fields have been confirmed, but scientists are actively continuing to research this area.

Unit - 1

Electrostatics - I

Vector Dot Product

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

* Projection of $\vec{A}$ on $\vec{B} \Rightarrow |\vec{A}|\cos\theta$

Projection of $\vec{B}$ on $\vec{A} \Rightarrow |\vec{B}|\cos\theta$

* If θ is not given, then

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Given
$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$
$\vec{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$

* To find the angle between two vectors, then

$$\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

Vector Cross Product

$$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\,\hat{a}_n$$

* If two vectors are parallel, then

$$\vec{A}\times\vec{B} = 0$$

* If θ is not given, then

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Given
$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$
$\vec{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$

[or]

$$\theta = \sin^{-1}\frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}$$

Co-ordinate Systems

Cartesian Co-ordinate System

Co-ordinates

x, y, z

Position Vector

$$\vec{R}_{OP} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

Unit Vectors:

$\hat{a}_x, \hat{a}_y, \hat{a}_z$

$$\hat{R}_{OP} = \frac{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}{\sqrt{x^2+y^2+z^2}}$$

Limits

$x \Rightarrow -\infty$ to $+\infty$

$y \Rightarrow -\infty$ to $+\infty$

$z \Rightarrow -\infty$ to $+\infty$

Differential Length

$$d\vec{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

Differential Volume:

$$dv = dx\,dy\,dz$$

Differential Surface:

$d\vec{S}_x = dy\,dz\,\hat{a}_x$

$d\vec{S}_y = dx\,dz\,\hat{a}_y$

$d\vec{S}_z = dx\,dy\,\hat{a}_z$

Cylindrical Co-ordinate System

Co-ordinates

ρ, ϕ, z

Position Vector:

$$\vec{R}_{OP} = A_\rho\hat{a}_\rho + A_\phi\hat{a}_\phi + A_z\hat{a}_z$$

Unit Vectors:

$\hat{a}_\rho, \hat{a}_\phi, \hat{a}_z$

$$\hat{R}_{OP} = \frac{A_\rho\hat{a}_\rho + A_\phi\hat{a}_\phi + A_z\hat{a}_z}{\sqrt{A_\rho^2 + A_\phi^2 + A_z^2}}$$

Limits

$\rho \Rightarrow 0$ to ∞

$\phi \Rightarrow 0$ to 2π

$z \Rightarrow -\infty$ to ∞

Differential Length

$$d\vec{l} = d\rho\,\hat{a}_\rho + \rho\,d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$$

Differential Volume

$$dv = \rho\,d\rho\,d\phi\,dz$$

Differential Surface:

$d\vec{S}_\rho = \rho\,d\phi\,dz\,\hat{a}_\rho$

$d\vec{S}_\phi = d\rho\,dz\,\hat{a}_\phi$

$d\vec{S}_z = \rho\,d\rho\,d\phi\,\hat{a}_z$

Spherical Co-ordinate System

Co-ordinates

r, θ, ϕ

Position Vector

$$\vec{R}_{OP} = A_r\hat{a}_r + A_\phi\hat{a}_\phi + A_\theta\hat{a}_\theta$$

Unit Vectors:

$\hat{a}_r, \hat{a}_\phi, \hat{a}_\theta$

$$\hat{R}_{OP} = \frac{A_r\hat{a}_r + A_\phi\hat{a}_\phi + A_\theta\hat{a}_\theta}{\sqrt{A_r^2 + A_\phi^2 + A_\theta^2}}$$

Limits

$r \Rightarrow 0$ to ∞

$\phi \Rightarrow 0$ to 2π (360°)

$\theta \Rightarrow 0$ to π (180°)

Differential Length

$$d\vec{l} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

Differential Volume

$$dv = r^2\sin\theta\,dr\,d\theta\,d\phi$$

Differential Surface

$d\vec{S}_r = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r$

$d\vec{S}_\theta = r\sin\theta\,dr\,d\phi\,\hat{a}_\theta$

$d\vec{S}_\phi = r\,dr\,d\theta\,\hat{a}_\phi$

Transformation of Co-ordinates:

Cartesian to Cylindrical

$\{x, y, z \Rightarrow \rho, \phi, z\}$

$$\rho = \sqrt{x^2+y^2}$$

$$\phi = \tan^{-1}\frac{y}{x} \; ; \; \phi = \cos^{-1}\frac{x}{\rho}$$

$$z = z$$

Cylindrical to Cartesian

$\{\rho, \phi, z \Rightarrow x, y, z\}$

$$x = \rho\cos\phi$$

$$y = \rho\sin\phi$$

$$z = z$$

Cartesian to Spherical

$\{x, y, z \Rightarrow r, \theta, \phi\}$

$$r = \sqrt{x^2+y^2+z^2}$$

$$\theta = \cos^{-1}\frac{z}{r} \; \text{(or)}$$

$$\theta = \tan^{-1}\frac{\rho}{z} = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z}$$

$$\phi = \tan^{-1}\frac{y}{x} \; \text{[or]}$$

$$\phi = \cos^{-1}\frac{x}{\sqrt{x^2+y^2}}$$

Spherical to Cartesian

$\{r, \theta, \phi \Rightarrow x, y, z\}$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$\Rightarrow \sin\theta = \rho/r$

Transformation of Vectors:

Cartesian to Cylindrical

$\{A_x, A_y, A_z \Rightarrow A_\rho, A_\phi, A_z\}$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Cylindrical to Cartesian

$\{A_\rho, A_\phi, A_z\} \Rightarrow \{A_x, A_y, A_z\}$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Cartesian to Spherical

$\{A_x, A_y, A_z \Rightarrow A_r, A_\theta, A_\phi\}$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Spherical to Cartesian

$\{A_r, A_\theta, A_\phi \Rightarrow A_x, A_y, A_z\}$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Spherical to Cylindrical

$\{A_r, A_\theta, A_\phi \Rightarrow A_\rho, A_\phi, A_z\}$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Cylindrical to Spherical

$\{A_\rho, A_\phi, A_z \Rightarrow A_r, A_\theta, A_\phi\}$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Vector Calculus:

Gradient:

The gradient of a scalar field 'V' is a vector that represents both the magnitude and the direction of the maximum space rate of increase of 'V'

Cartesian Coordinates

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

Cylindrical Coordinates

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z$$

Spherical Coordinates

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi$$

Divergence:

The divergence of '$\vec{A}$' at a given point 'P' is the outward flux per unit Volume as the Volume shrinks about 'P'

$$\operatorname{div}\vec{A} = \nabla\cdot\vec{A} = \lim_{\Delta v \to 0}\frac{\oint_S \vec{A}\cdot d\vec{s}}{\Delta v}$$

Cartesian

$$\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cylindrical

$$\nabla\cdot\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Spherical

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl:

The curl of $\vec{A}$ is an axial (or rotational) vector whose magnitude is the maximum circulation of $\vec{A}$ per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maximum.

$$\operatorname{curl}\vec{A} = \nabla\times\vec{A} = \left(\lim_{\Delta s \to 0}\frac{\oint_L \vec{A}\cdot d\vec{l}}{\Delta s}\right)_{max}\hat{a}_n$$

Divergence Theorem

$$\oint_S \vec{A}\cdot d\vec{s} = \int_v \nabla\cdot\vec{A}\, dv$$

Relation between surface integral and Volume integral

Stokes Theorem

$$\oint_L \vec{A}\cdot d\vec{l} = \int_S (\nabla\times\vec{A})\cdot d\vec{S}$$

Relation between line integral and surface integral.

Cartesian

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Cylindrical

$$\nabla\times\vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

Spherical

$$\nabla\times\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix}$$

DIVERGENCE THEOREM

Gauss's divergence theorem states that " the volume integral of the divergence of a vector field $\vec{A}$ taken over any volume 'v' is equal to the surface integral of $\vec{A}$ taken over the closed surface surrounding the volume 'v'."

$$\iiint_V (\nabla \cdot \vec{A})\, dv = \iint_S \vec{A} \cdot d\vec{S}$$

PROOF

$$\iiint_V (\nabla \cdot \vec{A})\, dv = \iiint \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) dx\, dy\, dz$$

$$= \iiint_V \frac{\partial A_x}{\partial x} dx\, dy\, dz + \iiint_V \frac{\partial A_y}{\partial y} dx\, dy\, dz + \iiint_V \frac{\partial A_z}{\partial z} dx\, dy\, dz$$

Let us consider only the first integral

$$\iiint_V \frac{\partial A_x}{\partial x} dx\, dy\, dz = \iint \left[\int \frac{\partial A_x}{\partial x} dx\right] dy\, dz = \iint_S A_x\, dy\, dz$$

Similarly

$$= \iint_S A_x\, dy\, dz + \iint_S A_y\, dx\, dz + \iint_S A_z\, dx\, dy$$

$$= \iint_S A_x\, dS_x + \iint_S A_y\, dS_y + \iint_S A_z\, dS_z$$

WKT

$d\vec{S_x} = dy\, dz\, \hat{a}_x$

$d\vec{S_y} = dx\, dz\, \hat{a}_y$

$d\vec{S_z} = dx\, dy\, \hat{a}_z$

By splitting the above integral.

$$= \iint_S \underbrace{[A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z]}_{\vec{A}} \underbrace{[dS_x \hat{a}_x + dS_y \hat{a}_y + dS_z \hat{a}_z]}_{d\vec{S}}$$

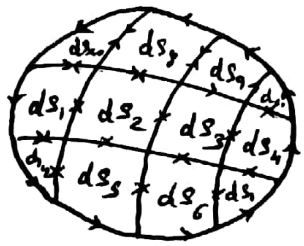
$$\therefore \iiint_V (\nabla \cdot \vec{A})\, dv = \iint_S \vec{A} \cdot d\vec{S}$$

Hence proved.

STOKES THEOREM

Stokes theorem states that "the surface integral of the curl of a vector field $\vec{A}$ taken over any surface 'S' is equal to the line integral of $\vec{A}$ around the closed poriphory of the surface.

$$\therefore \boxed{\oint_L \vec{A}\cdot d\vec{l} = \iint_S (\nabla\times\vec{A})\cdot d\vec{s}}$$



PROOF

$$\oint_L \vec{A}\cdot d\vec{l} = \oint_{ds_1} \vec{A}\cdot d\vec{l} + \oint_{ds_2} \vec{A}\cdot d\vec{l} + \oint_{ds_3} \vec{A}\cdot d\vec{l} + \cdots$$

According to Curl formula,

$$\nabla\times\vec{A} = \left[\lim_{ds\to 0} \frac{\oint_L \vec{A}\cdot d\vec{l}}{ds}\right]\hat{a}_n$$

By Rearranging

$$\oint_L \vec{A}\cdot d\vec{l} = (\nabla\times\vec{A})\cdot ds\,\hat{a}_n$$

$$= (\nabla\times\vec{A})\cdot d\vec{s}$$

$$\therefore \oint_L \vec{A}\cdot d\vec{l} = (\nabla\times\vec{A})\cdot d\vec{s}_1 + (\nabla\times\vec{A})\cdot d\vec{s}_2 + (\nabla\times\vec{A})\cdot d\vec{s}_3 + \cdots$$

Putting it into integral.

$$\boxed{\oint_L \vec{A}\cdot d\vec{l} = \iint_S (\nabla\times\vec{A})\cdot d\vec{s}}$$

Hence proved.

RELATION BETWEEN FIELD THEORY AND CIRCUIT THEORY

CIRCUIT THEORY	FIELD THEORY
1. Deals with voltage (V) and Current (I)	Deals with Electric (E) and magnetic (H) fields
2. V and I are scalars	$\vec{E}$ and $\vec{H}$ are vectors
3. V and I are produced from $\vec{E}$ and $\vec{H}$ respectively	$\vec{E}$ and $\vec{H}$ are produced from V and I respectively.
4. V and I are functions of time (t)	$\vec{E}$ and $\vec{H}$ are functions of time (t) and space variables (x, y, z) or (ρ, ϕ, z) or (r, θ, ϕ)
5. Radiation effects are neglected	Radiation effect can be considered.
6. Using circuit theory, transmitter and receiver circuits can be analyzed and designed. But it can not be analyzed and designed a medium like free space.	Using field theory any medium can be designed and analysed
7. This is simplified approximation of field theory	This is more accurate theory
8. Circuit theory can not be used to design or analyse a complete communication system	Field theory can be effectively used to design communication system.

CIRCUIT THEORY	FIELD THEORY
9. It is useful at low frequency.	It is useful at all frequencies, particularly at high frequencies
10. Basic laws are ohms law kirchoff's law	Basic laws are Coulomb's law, Gauss's law, Ampere's circuit law
11) Basic theorems are Thevenin's, Norton's, Reciprocity superposition Maximum Power transfer theorem.	Basic theorems are Reciprocity, Helmholtz, Stokes Divergence and Poynting theorem.
12) Basic equations are Mesh/loop equations	Basic equations are Maxwell, Poissons, Laplace and wave equations.
13) It is simple	It is complex but it can be made easy using appropriate mathematics

— X —

Coulomb's law states that the force F between two point charges Q_1 and Q_2 along the line joining them is

i) Directly proportional to the product of the two charges (Q_1, Q_2)

ii) Inversely proportional to the square of the distance between them (R)

Expressed mathematically as,

$$F \propto \frac{Q_1 Q_2}{R^2}$$

The proportionality can be replaced by proportionality constant k

$$\therefore \quad F = k \frac{Q_1 Q_2}{R^2}$$

where $k = \frac{1}{4\pi\varepsilon}$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

where ε_0 = Permittivity of free space $= 8.854 \times 10^{-12}$ F/m

ε_r = Relative ~~Permeability~~ Permittivity of medium, for air $\varepsilon_r = 1$

ε = Absolute permittivity

∴ The force between the two point charges located in free space or vacuum is given by,

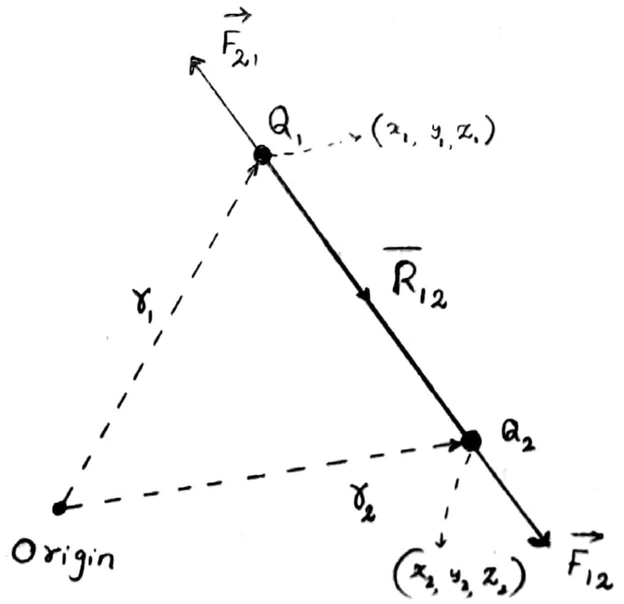
$$F = \frac{1}{4\pi \varepsilon_0 \varepsilon_r} \frac{Q_1 Q_2}{R^2}$$

$$\boxed{F = \frac{1}{4\pi \varepsilon_0} \frac{Q_1 Q_2}{R^2}}$$ $\boxed{\varepsilon_r = 1}$

VECTOR FORM OF COULOMB'S LAW

Force is the vector quantity. Coulombs Vector force on point charge Q_1, Q_2 is derived as follows.

* Consider a two point charge placed at a distance r_1 and r_2 from the origin.



COULOMB'S LAW OF FORCE VECTOR FORM

* The force due to Q_1 on Q_2 is designated as $\bar{F}_{12}$, here the charge is located at point 1 (Source point) and the Force is required at Point 2 (Field Point).

* The force due to Q_2 on Q_1 is designated as $\bar{F}_{21}$, here the charge is located at Point 2 (Source point) and the Force is required at Point 1 (Field point).

$$\therefore \quad \boxed{\bar{F}_{12} = k \frac{Q_1 Q_2}{|R_{12}|^2} \hat{a}_{12}}$$

where $\hat{a}_{12}$ is the unit vector along the direction $\bar{R}_{12}$

$$\therefore \quad \hat{a}_{12} = \frac{\vec{R}_{12}}{|\bar{R}_{12}|} = \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|}$$

$$\bar{F}_{12} = k \frac{Q_1 Q_2}{|R_{12}|^2} \frac{\bar{R}_{12}}{|R_{12}|}$$

$$\boxed{\bar{F}_{12} = k \frac{Q_1 Q_2}{|R_{12}|^3} \bar{R}_{12}}$$

ELECTRIC FIELD INTENSITY:

"Electric field intensity (electric field strength) is the force per unit charge when placed in a electric field"

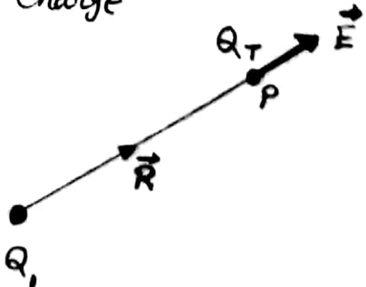
$$\boxed{\bar{E} = \frac{\bar{F}}{Q}}$$

* It's unit can be newton/coulomb (or) volt/metre
* Electric field intensity is the vector quantity and it acts in the direction as Coulombs law of force
* Electric field intensity due to charge 'Q_1' is given by

$$\bar{E} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} \hat{a}_R \times \frac{1}{Q_1}$$

$\therefore \bar{E} =$

* Electric field intensity due to point charge 'Q_1' at point 'P' is given by



$$\bar{E} = \frac{\text{Force on test charge}}{\text{Test charge}} = \frac{F_t}{Q_t}$$

WKT,

$$\vec{F}_T = \frac{Q_1 Q_T}{4\pi \epsilon_0 |R^2|} \hat{a}_R$$

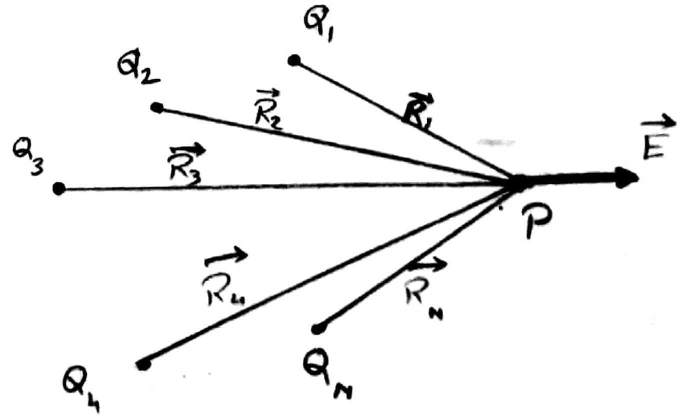
$$\vec{E} = \frac{1}{Q_T} \cdot \frac{Q_1 Q_T}{4\pi \epsilon_0 |R^2|} \hat{a}_R$$

$$\hat{a}_r = \frac{\bar{r}_p - \bar{r}_1}{|\bar{r}_p - \bar{r}_1|}$$

$$\boxed{\vec{E} = \frac{Q_1}{4\pi \epsilon_0 |R^2|} \hat{a}_R \quad N/C \text{ (or) } V/m}$$

If there are 'N' point charges $Q_1, Q_2, \ldots Q_N$ located at $r_1, r_2, \ldots r_N$, then the electric field intensity at point 'P' is obtained from

$$\boxed{\vec{E} = \frac{1}{4\pi \epsilon_0} \sum_{i=1}^{N} \frac{Q_i (\bar{r}_p - \bar{r}_i)}{|\bar{r}_p - \bar{r}_i|^3}}$$



ELECTRIC FIELD DUE TO CONTINUOUS CHARGE DISTRIBUTION

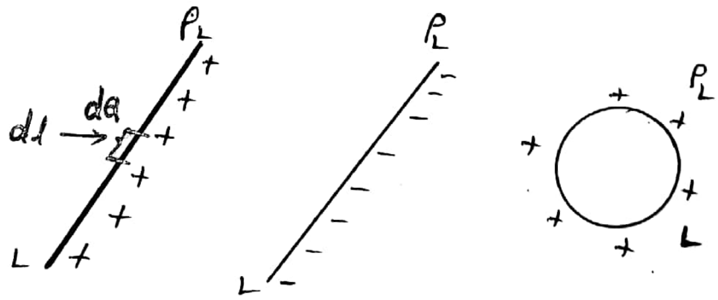
* Continuous charge distribution includes

(i) Line charge distribution

(ii) Surface charge distribution

(iii) Volume charge distribution.

LINE CHARGE DISTRIBUTION:



* If the distribution of charge along the line then it is called line charge distribution.

* Since, the charges are distributed, it is customary to note the charge density (line charge density) 'ρ_L'

$$\boxed{\rho_L = \frac{\text{Total charge spread}}{\text{Length of line}}}$$

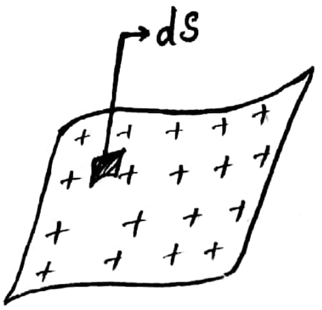
$$\boxed{\bar{E} = \frac{\int \rho_L \, dl}{4\pi\epsilon_0 R^2} \, \hat{a}_R}$$

* If we consider a differential length 'dl'

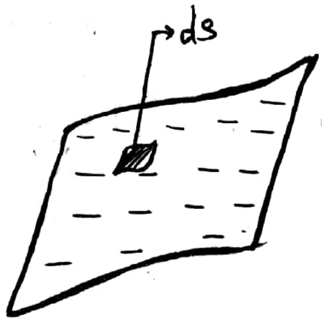
then, $\boxed{dQ = \rho_L \, dl} \rightarrow \boxed{Q = \int \rho_L \, dl}$

SURFACE CHARGE DISTRIBUTION:

* When the charge is uniformly spread over a surface then that distribution is called surface charge distribution



SURFACE CHARGE
(Positive charge is distributed)



(Negative charge is distributed)

* Here surface charge density 'ρ_s' is given by

$$\rho_s = \frac{\text{Total charge spread}}{\text{Area of surface}} \quad C/m^2$$

* Now,

$$dQ = \rho_s \, ds$$

where
- $ds \rightarrow$ differential surface area
- $\rho_s \rightarrow$ surface charge density
- $dQ \rightarrow$ charge in the differential surface area

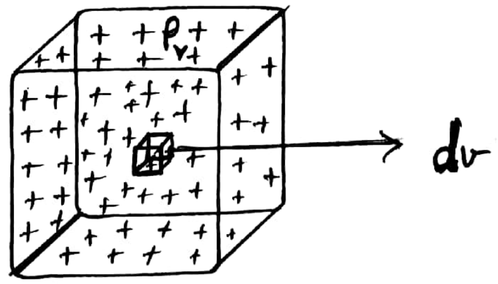
∴ Total charge in the surface area can be found from,

$$Q = \int_S \rho_s \, ds$$

* Electric field intensity due to surface charge $\bar{E} = \dfrac{\int \rho_s \, ds}{4\pi\varepsilon_0 R^2} \hat{a}_R$

VOLUME CHARGE DISTRIBUTION

* Here the charge is uniformly filled in the volume



Volume charge distribution

* The volume charge density, ρ_v is given by

$$\rho_v = \frac{\text{charge filled in the volume}}{\text{Volume of the element}} \quad C/m^3$$

* Now,

$$\boxed{dQ = \rho_v \, dv} \qquad \because \rho_v = \frac{Q}{V}$$

* where dv = differential volume
  ρ_v = Volume charge density
  dQ = charge in differential volume.

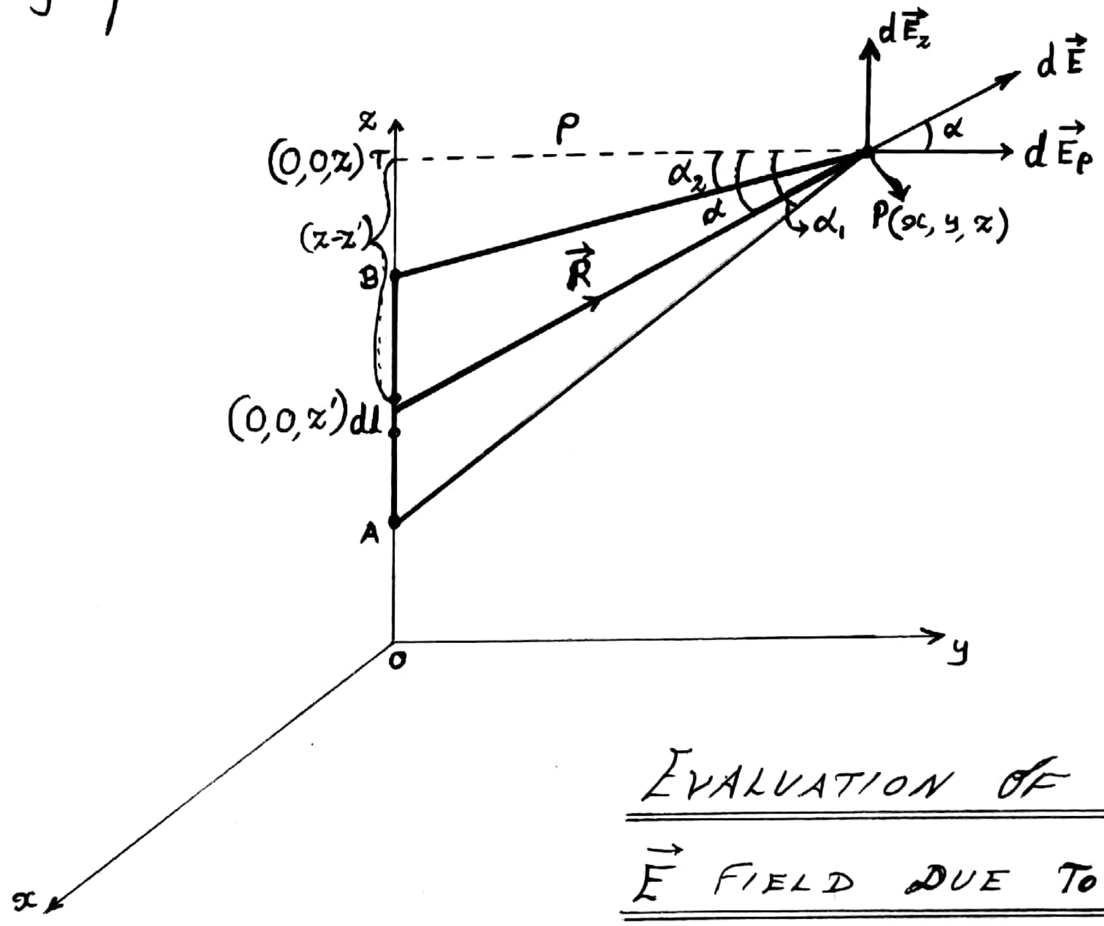
$\therefore$ Total charge in the volume can be found from,

$$\boxed{Q = \int_V \rho_v \, dv}$$

* Electric field intensity due to volume charge, $\boxed{\vec{E} = \frac{\int_V \rho_v \, dv}{4\pi\varepsilon_0 R^2} \hat{a}_R}$

Electric Field Intensity ($\vec{E}$) Due To Line Charge Distribution

* Consider the line charge with uniform charge density (ρ_L) extending from A to B along the z-axis.



Evaluation of the $\vec{E}$ Field Due To A Line Charge

* Consider a small differential length[dl] along the line charge 'AB'.
* 'dQ' is the charge element ~~along~~ associated with 'dl'

WKT,

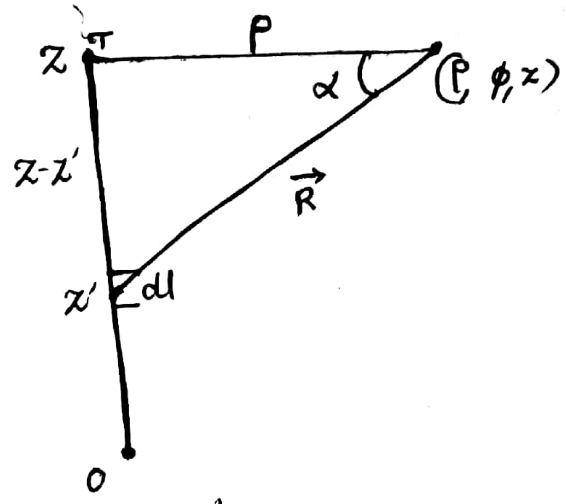
$$dQ = \rho_L \, dl$$

But, dl is along 'z' axis, so $dl = dz'$

$$\therefore \boxed{dQ = \rho_L \, dz'}$$

$\vec{E}$ for infinite line charge (ALITER)

Diagram → refer notes



$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\hat{a}_R \quad ; \quad dQ = \rho_L dz'$$

$$\left\{ \because \hat{a}_R = \frac{\vec{R}}{|\vec{R}|} \right.$$

$$d\vec{E} = \frac{\rho_L dz'}{4\pi\varepsilon_0 R^2}\hat{a}_R \quad ; \quad d\vec{E} = \frac{\rho_L dz'}{4\pi\varepsilon_0}\frac{\vec{R}}{R^3}$$

From triangle

$$\tan\alpha = \frac{z-z'}{\rho} \quad ; \quad z-z' = \rho\tan\alpha$$

$$\cos\alpha = \frac{\rho}{|\vec{R}|} \quad ; \quad |\vec{R}| = \rho\sec\alpha$$

$$z' = OT - (z-z')$$

$$z' = OT - \rho\tan\alpha$$

$$dz' = -\rho\sec^2\alpha\, d\alpha$$

$$\vec{R} = \rho\hat{a}_\rho + (z-z')\hat{a}_z$$

$$|\vec{R}| = \sqrt{\rho^2 + (z-z')^2}$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{z_A}^{z_B}\frac{\vec{R}}{R^3}dz'$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\frac{\rho\hat{a}_\rho + (z-z')\hat{a}_z}{(\rho\sec\alpha)^3}\left[-\rho\sec^2\alpha\, d\alpha\right]$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\frac{\rho\hat{a}_\rho + \rho\tan\alpha\,\hat{a}_z}{(\rho\sec\alpha)^3}(-\rho\sec^2\alpha\, d\alpha)$$

$$\vec{E} = -\frac{\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\frac{\rho\hat{a}_\rho + \rho\sin\alpha\sec\alpha\,\hat{a}_z}{(\rho\sec\alpha)^3}(\rho\sec^2\alpha\, d\alpha)$$

$$\vec{E} = -\frac{\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\frac{\rho\sec\alpha\left\{\frac{1}{\sec\alpha}\hat{a}_\rho + \sin\alpha\,\hat{a}_z\right\}}{(\rho\sec\alpha)^3}\rho\sec^2\alpha\, d\alpha$$

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0\rho}\int_{\alpha_1}^{\alpha_2}(\cos\alpha\,\hat{a}_\rho + \sin\alpha\,\hat{a}_z)\,d\alpha$$

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left\{\left[\sin\alpha\right]_{\alpha_1}^{\alpha_2}\hat{a}_\rho + \left[-\cos\alpha\right]_{\alpha_1}^{\alpha_2}\right\}$$

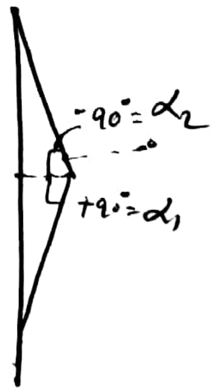
$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left\{(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\rho - (\cos\alpha_2 - \cos\alpha_1)\hat{a}_z\right\}$$

$$\boxed{\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0\rho}\left\{-(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\hat{a}_z\right\}}$$

The above relation is called $\vec{E}$ for ~~point~~ finite line charge.

SPECIAL CASE: [For Infinite line charge]

For infinite line charge, $\alpha_1 = 90^\circ$; $\alpha_2 = -90^\circ$



$$\therefore \vec{E} = \frac{\rho_L}{4\pi\varepsilon_0\rho}\left[-\left[\underbrace{\sin(-90^\circ)}_{-1} - \underbrace{\sin 90^\circ}_{-1}\right]\right]\hat{a}_\rho$$

$$\underbrace{-(-2)}_{2}$$

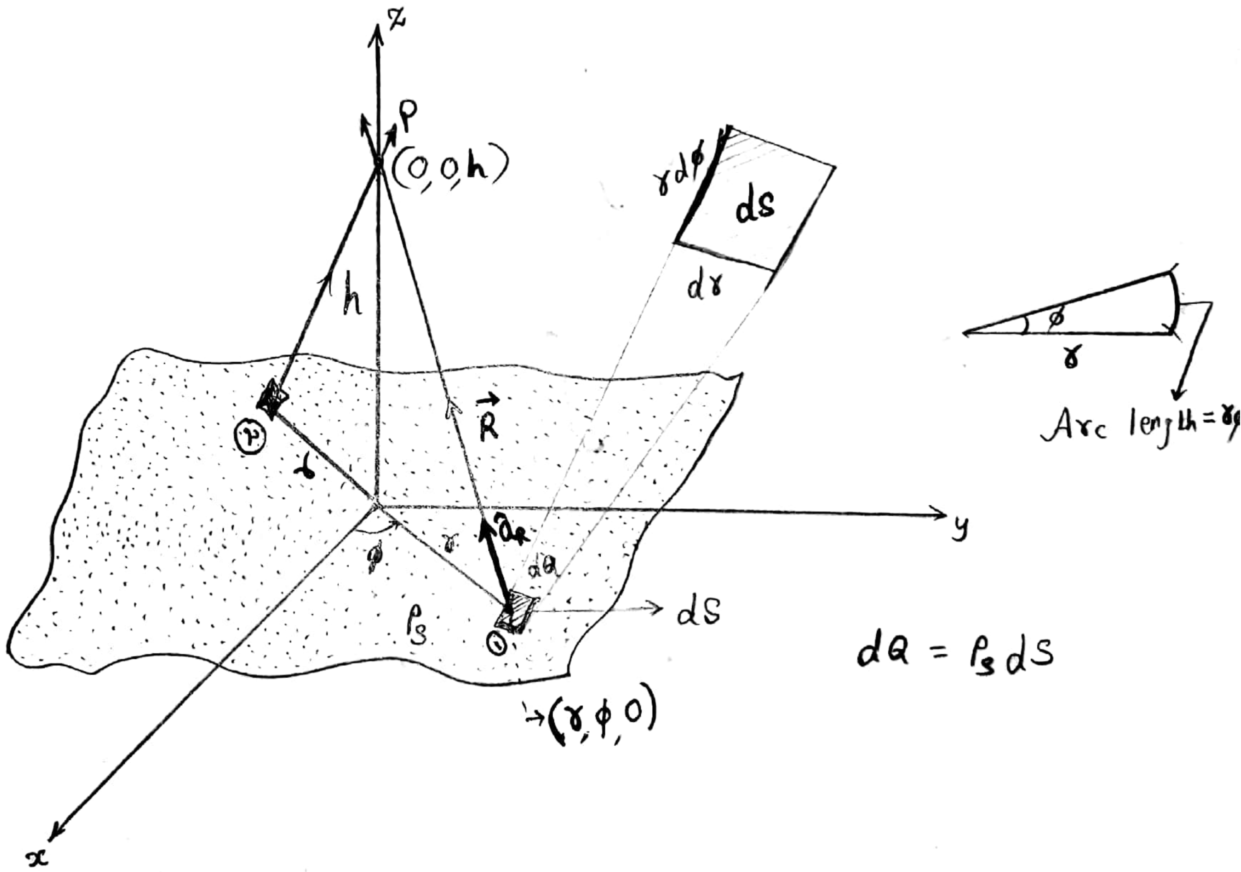
$$\therefore \vec{E} = \frac{\rho_L}{\frac{4\pi}{2}\varepsilon_0\rho}(2)$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho}$$

ELECTRIC FIELD DUE TO INFINITE SHEET OF CHARGE

Consider a infinite sheet of charge in the x-y plane with uniform charge density ρ_s. The charge associated with an elemental area dS is (17)

$$\boxed{dQ = \rho_s \, dS} \quad \text{— (1)}$$



where

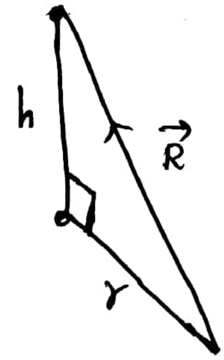
$$dS = (r\,d\phi)\,dr$$

$$\therefore \quad dQ = \rho_s \, r \, d\phi \, dr \quad \text{— (2)}$$

The electric field intensity at point P due to the differential element dS is given by,

$\vec{E}$ for infinite sheet of charge (ALITER)

Refer diagram from notes.



$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\hat{a}_R \quad ; \quad \hat{a}_R = \frac{\vec{R}}{|\vec{R}|}$$

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^3}\vec{R} \quad ; \quad dQ = \rho_s ds$$

$\vec{R} = r\hat{a}_r + h\hat{a}_z$

In $\hat{a}_r$ direction field is constant

$\therefore \vec{R} = h\hat{a}_z$

$|\vec{R}| = \sqrt{r^2+h^2}$

$$d\vec{E} = \frac{\rho_s ds}{4\pi\varepsilon_0 (r^2+h^2)^{3/2}} h\hat{a}_z \quad \Big| \quad dQ = \rho_s r\,d\phi\,dr$$

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0}\int_s \frac{r\,d\phi\,dr}{(r^2+h^2)^{3/2}}\hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0}\int_{\phi=0}^{2\pi} d\phi \int_{r=0}^{\infty}\frac{r\,dr}{(r^2+h^2)^{3/2}}\hat{a}_z$$

Let

$u^2 = r^2 + h^2$

$2u\,du = 2r\,dr$

$u\,du = r\,dr$

$r = 0 ; \; u = h$

$r = \infty \quad u = \infty$

$$\therefore \vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0}(2\pi)\int_{r=0}\frac{u\,du}{u^3}\hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0}\int_{u=h}^{\infty}\frac{1}{u^2}du\;\hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0}\left[-\frac{1}{u}\right]_h^{\infty}\hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0}\left[-\frac{1}{\infty} + \frac{1}{h}\right]\hat{a}_z \quad \Big| \quad \vec{E} = \frac{\rho_s h}{2\varepsilon_0}\left(\frac{1}{h}\right)\hat{a}_z$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_z}$$

$$\therefore \boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0} \hat{a}_n}$$

$\because \hat{a}_z$ is normal to surface.

For parallel plate capacitor.

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$\vec{E}_- = \frac{-\rho_s}{2\varepsilon_0} (-\hat{a}_n)$$

$$\therefore \vec{E} = \frac{\rho_s}{2\varepsilon_0} \hat{a}_n + \frac{\rho_s}{2\varepsilon_0} \hat{a}_n$$

$$\boxed{\vec{E} = \frac{\rho_s}{\varepsilon_0} \hat{a}_n}$$

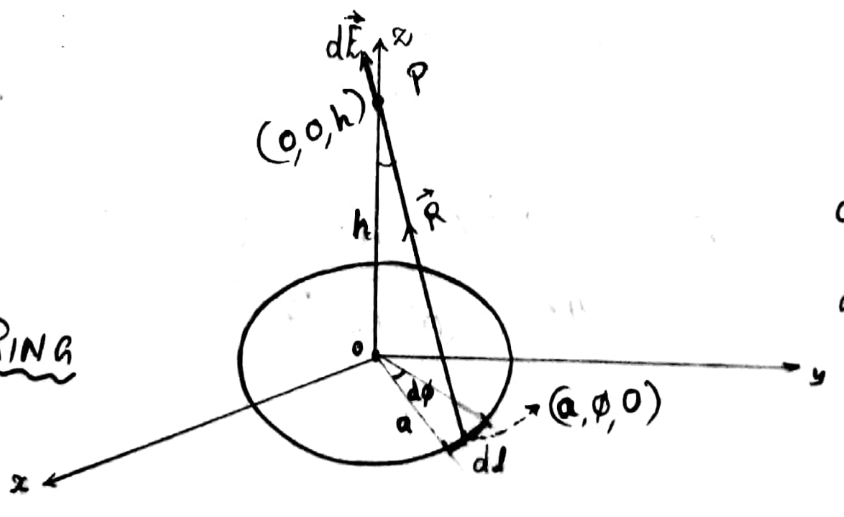
INFERENCE:

* Electric field intensity is normal to the sheet
* Electric field is independent of the distance between the sheet and the point of observation.

ELECTRIC FIELD INTENSITY DUE TO CIRCULAR RING OF RADIUS 'a'

* Consider a ring of charge density ρ_L placed on 'x y' plane.
* Consider a differential element 'dl' on the ring.
* The charge associated with differential element 'dl' is given by,

$$\boxed{dQ = \rho_L\, dl} \quad -①$$



CHARGED RING

$dQ = \rho_L\, dl$

$dl = a\, d\phi$

Electric field intensity due to differential element 'dl' is given by

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\,\hat{a}_R$$

where $dQ = \rho_L\, dl = \rho_L\, a\, d\phi$

$$\therefore \quad d\vec{E} = \frac{\rho_L\, a\, d\phi}{4\pi\varepsilon_0 R^2}\,\hat{a}_R$$

Now,

$$\hat{a}_R = \frac{r_p - r_l}{|r_p - r_l|} = \frac{\overbrace{(0-a)\hat{a}_r + (0-\phi)\hat{a}_\phi}^{\text{Canceled}} + (h-0)\hat{a}_z}{\sqrt{a^2+h^2}}$$

* Along the direction ϕ, field is constant so 'ϕ' get canceled

* Along the radial direction r, there will be equal charge, which will cancel each other.

* The only component available is $\hat{a}_z$

$$\therefore \quad \hat{a}_R = \frac{h\,\hat{a}_z}{\sqrt{a^2+h^2}}$$

$$R^2 = a^2 + h^2$$

$$\therefore \quad d\vec{E} = \frac{\rho_L\, a\, d\phi\; h}{4\pi\varepsilon_0 (a^2+h^2)^{3/2}}\,\hat{a}_z$$

To find total field intensity due to entire ring is obtained by integrating the above equation w.r.to ϕ (25)

ϕ varies between → 0 to 2π

$$\therefore \vec{E} = \frac{\rho_L a h}{4\pi \varepsilon_0 (a^2+h^2)^{3/2}} \int_0^{2\pi} d\phi \; \hat{a}_z$$

$$\vec{E} = \frac{\rho_L a h}{\frac{4\pi}{2} \varepsilon_0 (a^2+h^2)^{3/2}} 2\pi \; \hat{a}_z$$

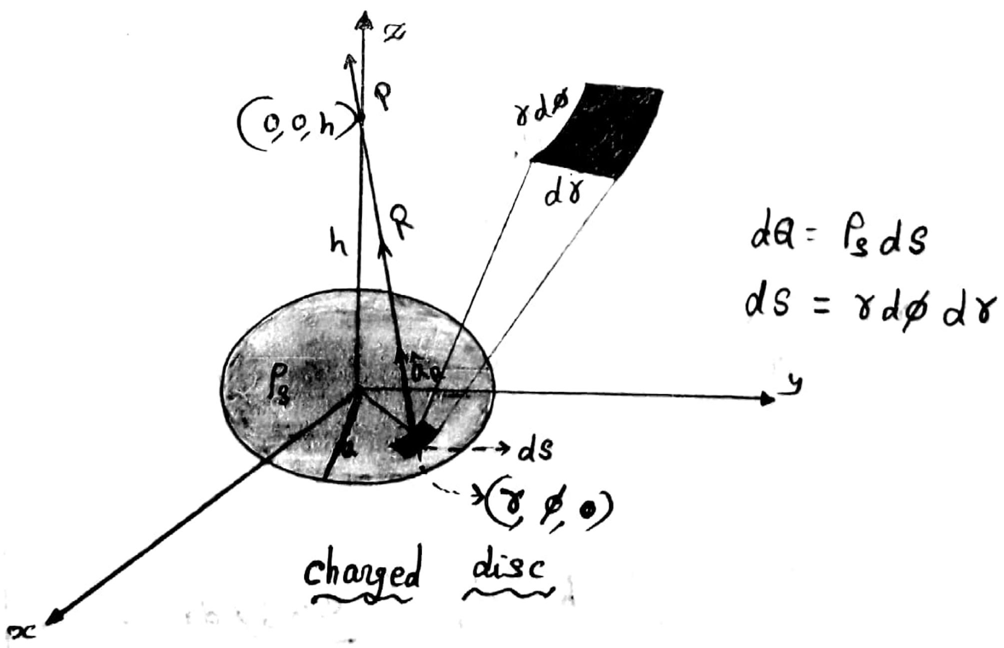
$$\boxed{\vec{E} = \frac{\rho_L a h}{2 \varepsilon_0 [h^2+a^2]^{3/2}} \hat{a}_z}$$

ELECTRIC FIELD INTENSITY DUE TO CIRCULAR DISC OF RADIUS 'a' :

(27)

* Consider a circular disc of radia a, which has a uniform charge density of ρ_s C/m²
* The circular disc is placed on the x-y plane.
* Consider a differential element 'ds' on a disc
* Charge associated with this element 'ds' is given by

$$\boxed{dQ = \rho_s \, dS}$$



charged disc

Electric field intensity at point 'P' due to ~~an~~ differential element dS, is given by

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\,\hat{a}_R$$

$$d\vec{E} = \frac{\rho_s\, ds}{4\pi\varepsilon_0 R^2}\,\hat{a}_R$$

where,

$$ds = r\,d\phi\,dr$$

$$d\vec{E} = \frac{\rho_s\, r\,d\phi\,dr}{4\pi\varepsilon_0 R^2}\,\hat{a}_R$$

$$\hat{a}_R = \frac{r_p - r_s}{|r_p - r_s|} = \frac{-r\hat{a}_r + h\hat{a}_z}{\sqrt{r^2+h^2}}$$

Due to symmetry the radial component $(\hat{a}_r)$ got cancelled.

$$\therefore\ \hat{a}_R = \frac{h\,\hat{a}_z}{\sqrt{r^2+h^2}}$$

$$d\vec{E} = \frac{\rho_s\, r\,d\phi\,dr}{4\pi\varepsilon_0\,(r^2+h^2)^{3/2}}\,h\,\hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0}\int_{\phi=0}^{2\pi} d\phi \int_{r=0}^{a} \frac{r\,dr}{(r^2+h^2)^{3/2}}\,\hat{a}_z$$

ϕ varies between 0 and 2π

r varies between 0 and a

$$\vec{E} = \frac{\rho_s h}{\frac{4\pi\varepsilon_0}{2}}\,2\pi \int_0^a \frac{r\,dr}{(r^2+h^2)^{3/2}}\,\hat{a}_z$$

(29)

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0}\int_0^a \frac{r\,dr}{(r^2+h^2)^{3/2}}\,\hat{a}_z$$

Let, $u = (r^2+h^2)^{1/2} = \sqrt{r^2+h^2}$

$u^2 = r^2 + h^2$

$2u\,du = 2r\,dr + 0$

$r\,dr = u\,du$

when, $r = 0$

$u^2 = 0 + h^2$

$u = h$

when, $r = a$

$u^2 = a^2 + h^2$

$u = \sqrt{a^2+h^2}$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0}\int_h^{\sqrt{a^2+h^2}} \frac{u\,du}{u^{3/2}}$$

$$\int u^{-2}du = -\frac{1}{u}$$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0}\left[-\frac{1}{u}\right]_h^{\sqrt{a^2+h^2}}$$

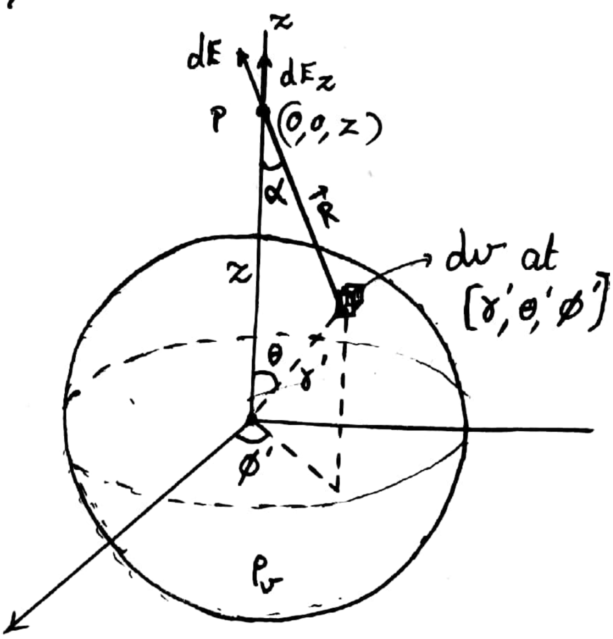
$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0}\left[+\frac{1}{h} - \frac{1}{\sqrt{a^2+h^2}}\right]$$

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\left[1 - \frac{h}{\sqrt{a^2+h^2}}\right]\hat{a}_z$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\left[1 - \frac{h}{\sqrt{a^2+h^2}}\right]\hat{a}_z}$$

ELECTRIC FIELD DUE TO VOLUME CHARGE DISTRIBUTION

* Let us consider a sphere of radius a centered at the origin.
* $\rho_v \Rightarrow$ Volume charge density.

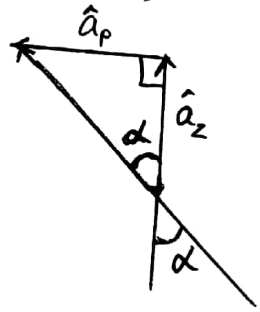


In general, $d\vec{E} = \dfrac{dQ}{4\pi\varepsilon_0 R^2}\,\hat{a}_R$

* Since $dQ = \rho_v\, dv$

$$\therefore \quad d\vec{E} = \frac{\rho_v\, dv}{4\pi\varepsilon_0 R^2}\,\hat{a}_R \quad \text{at point } P(0,0,z)$$

* Here $\hat{a}_R = \cos\alpha\,\hat{a}_z + \sin\alpha\,\hat{a}_\rho$



* Due to symmetricity along $\hat{a}_\rho$ direction the field component gets cancelled out.

$$\therefore \quad \hat{a}_R = \cos\alpha\,\hat{a}_z$$

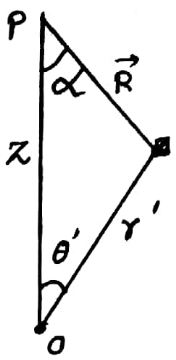
* Now, $$\vec{E} = \int_v \frac{\rho_v\, dv}{4\pi\varepsilon_0 R^2}\cos\alpha\,\hat{a}_z = \frac{\rho_v}{4\pi\varepsilon_0}\int_v \frac{dv\cos\alpha}{R^2}\,\hat{a}_z$$

where $dv = r'^2 \sin\theta'\, dr'\, d\theta'\, d\phi'$

$$\vec{E} = \frac{\rho_v}{4\pi\varepsilon_0}\int_v \frac{\cos\alpha}{R^2}\left[r'^2\sin\theta'\, dr'\, d\theta'\, d\phi'\right]\hat{a}_z$$

For integrating the above equation, few modification has to be done as follows.

Apply Cosine rule for the triangle given.



$$R^2 = z^2 + r'^2 - 2zr'\cos\theta$$

$$r'^2 = z^2 + R^2 - 2zR\cos\alpha$$

$$\cos\alpha = \frac{z^2 + R^2 - r'^2}{2zR} \; ; \quad \cos\theta' = \frac{z^2 + r'^2 - R^2}{2zr'}$$

we need $\sin\theta'$, $\therefore \; -\sin\theta' d\theta' = -\dfrac{2R\,dR}{2zr'}$

$$\sin\theta' d\theta' = \frac{2R\,dR}{2zr'}$$

; { R varies from $z-r'$ to $z+r'$, when θ' varies from 0 to π }

Now,

$$\vec{E} = \frac{\rho_v}{4\pi\varepsilon_0}\int_{\phi=0}^{2\pi} d\phi \int_{r'=0}^{a}\int_{R=z-r'}^{z+r'} \left[\frac{z^2 + R^2 - r'^2}{2zR\cdot R^2}\right] r'^2 \frac{R\,dR}{z r'}\, dr'$$

$$\vec{E} = \frac{\rho_v}{4\pi\varepsilon_0}[2\pi]\int_{r'=0}^{a}\int_{R=z-r'}^{z+r'}\left[\frac{z^2+R^2-r'^2}{2zR^2}\right] r'\left[\frac{dR}{z}\right] dr'$$

$$\vec{E} = \frac{\rho_v}{4\varepsilon_0 z^2}\int_{r'=0}^{a} r'\,dr' \int_{R=z-r'}^{z+r'}\left[\frac{z^2+R^2-r'^2}{R^2}\right] dR$$

$$\vec{E} = \frac{\rho_v}{4\varepsilon_0 z^2}\int_{r'=0}^{a} r'\,dr' \int_{R=z-r'}^{z+r'} 1 + \frac{z^2 - r'^2}{R^2}\, dR$$

$$\vec{E} = \frac{\rho_v}{4\varepsilon_0 z^2}\int_{r'=0}^{a} r'\,dr' \left[R - \frac{(z^2 - r'^2)}{R}\right]_{z-r'}^{z+r'}$$

$$\vec{E} = \frac{\rho_v}{4\varepsilon_0 z^2}\int_{r'=0}^{a} r' \left[\frac{R^2 - z^2 + r'^2}{R}\right]_{z-r'}^{z+r'} dr'$$

$$\vec{E} = \frac{\rho_v}{4\varepsilon_0 z^2}\int_{r'=0}^{a} r' \left\{\frac{z^2 + r'^2 + 2zr' - z^2 + r'^2}{z + r'} - \frac{z^2 + r'^2 - 2zr' - z^2 + r'^2}{z - r'}\right\} dr'$$

$$\vec{E} = \frac{\rho_v}{4\varepsilon_0 z^2}\int_{r'=0}^{a} r' \left\{\frac{2r'^2 + 2zr'}{z + r'} - \frac{2r'^2 - 2zr'}{z - r'}\right\} dr'$$

$$\vec{E} = \frac{\rho_v}{4\varepsilon_0 z^2}\int_{r'=0}^{a} r' \left\{\frac{2r'^2 z + 2z^2 r' - 2r'^3 - 2zr'^2 - 2r'^2 z + 2z^2 r' - 2r'^3 + 2zr'^2}{z^2 - r'^2}\right\} dr'$$

$$\vec{E} = \frac{\rho_v}{4\varepsilon_0 z^2}\int_{r'=0}^{a} r' \, 4r' \left[\frac{z^2 - r'^2}{z^2 - r'^2}\right] dr'$$

$$\vec{E} = \frac{\rho_v}{4\varepsilon_0 z^2} 4\left[\frac{r'^3}{3}\right]_0^a = \frac{\rho_v}{4\varepsilon_0 z^2}\frac{4a^3}{3}\hat{a}_z$$

$\times$ & $\div$ by π

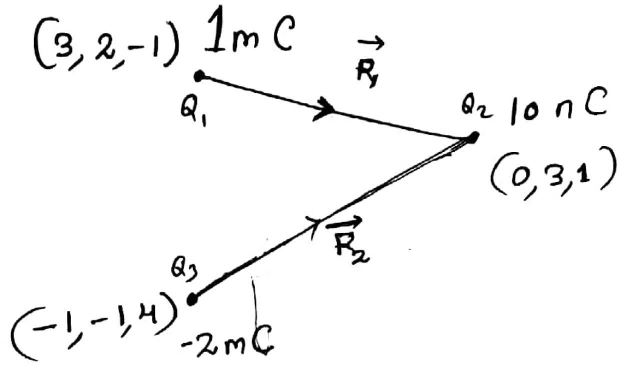
$$dQ = \rho_v \, dv$$
$$Q = \rho_v V$$

$$\therefore \vec{E} = \frac{\rho_v}{4\pi\varepsilon_0 z^2}\underbrace{\left\{\frac{4}{3}\pi a^3\right\}}_{\text{volume}}\hat{a}_z$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 z^2}\hat{a}_z \quad \text{or} \quad \boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R}$$

Point charges 1mC and -2mC are located at (3,2,-1) and (-1,-1,4) respectively. Calculate the electric force on an 10nC charge located at (0,3,1) and the electric field intensity at that point.

Solution



CASE 1

$$\vec{F}_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_1^3}\vec{R}_1$$

$\vec{R}_1 = (0-3)\hat{a}_x + (3-2)\hat{a}_y + (1+1)\hat{a}_z$

$\vec{R}_1 = -3\hat{a}_x + \hat{a}_y + 2\hat{a}_z$

$|\vec{R}| = \sqrt{3^2 + 1^2 + 2^2} = 3.74$

$$\vec{F}_1 = \frac{(1\times10^{-3})(10\times10^{-9})}{4\pi \times 8.854\times10^{-12}\times(3.74)^3}(-3\hat{a}_x + \hat{a}_y + 2\hat{a}_z)$$

$$\vec{F}_1 = [1.72\times10^{-3}](-3\hat{a}_x + \hat{a}_y + 2\hat{a}_z)$$

$$\vec{F}_1 = [-5.16\hat{a}_x + 1.72\hat{a}_y + 3.44\hat{a}_z]\times10^{-3}\ N$$

CASE 2

$$\vec{F}_2 = \frac{Q_3 Q_2}{4\pi\varepsilon_0 R_2^3}\vec{R}_2$$

$\vec{R}_2 = \hat{a}_x + 4\hat{a}_y - 3\hat{a}_z$

$|\vec{R}_2| = \sqrt{1^2 + 4^2 + (-3)^2} = 5.1$

$$\vec{F}_2 = \frac{(-2\times10^{-3})(10\times10^{-9})}{4\pi\times8.854\times10^{-12}\times(5.1)^3}(\hat{a}_x + 4\hat{a}_y - 3\hat{a}_z)$$

$$\vec{F}_2 = -(1.35 \times 10^{-3}) \quad \hat{a}_x + 4\hat{a}_y - 3\hat{a}_z$$

$$\vec{F}_2 = [-1.35\,\hat{a}_x - 5.42\,\hat{a}_y + 4.05\,\hat{a}_z] \times 10^{-3} \text{ N}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F} = (-5.16\,\hat{a}_x + 1.72\,\hat{a}_y + 3.44\,\hat{a}_z - 1.35\,\hat{a}_x - 5.42\,\hat{a}_y + 4.05\,\hat{a}_z) \times 10^{-3} \text{ N}$$

$$\boxed{\vec{F} = (-6.51\,\hat{a}_x - 3.7\,\hat{a}_y + 7.49\,\hat{a}_z) \times 10^{-3} \text{ N}}$$

$$|\vec{E}| = \frac{|\vec{F}|}{Q} \qquad |\vec{F}| = ~~10.2 \times 10^{-3} \text{ N}~~$$

$$10.59 \times 10^{-3} \text{ N}$$

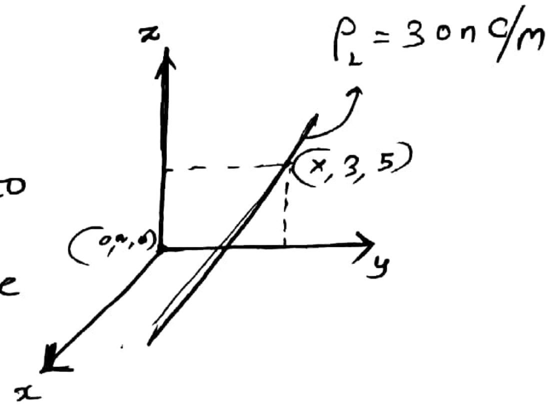
~~$|\vec{E}| = \frac{9.2 \times 10^{-3}}{10 \times 10^{-9}}$~~

~~$\boxed{|\vec{E}| = 920 \text{ kV/m}}$~~

$$|\vec{E}| = \frac{10.59 \times 10^{-3}}{10 \times 10^{-9}}$$

$$\boxed{|\vec{E}| = 1059.1} \text{ kV/m}$$

An infinitely long uniform line charge is located at $y = 3$; $z = 5$. If $\rho_L = 30\, nC/m$, find the field intensity $\vec{E}$ at origin



Solution

Since line charge is parallel to x-axis, no $\vec{E}$ component will be present along x-axis

WKT,

$$\vec{E} = \frac{\rho_L}{2\pi \varepsilon_0 r} \hat{a}_r$$

$$\hat{a}_r = \frac{\vec{R}}{|\vec{R}|} = \frac{-3\hat{a}_y - 5\hat{a}_z}{\sqrt{34}}$$

$$\vec{E} = \frac{30 \times 10^{-9}}{2\pi \times 8.854 \times 10^{-12} (\sqrt{34})^2} (-3\hat{a}_y - 5\hat{a}_z)$$

$$\boxed{\vec{E} = -47.58\, \hat{a}_y - 79.3\, \hat{a}_z} \; V/m$$

Two identical Conducting spheres have charges of 2×10^{-9} C and -0.5×10^{-9} C. when they are brought into Contact and then separated by 4 cm, what is the force between them?

solution

$$Q = \frac{Q_1 + Q_2}{2} = \frac{2\times10^{-9} - 0.5\times10^{-9}}{2}$$

$$Q_1 = Q_2 = Q = 0.75 \times 10^{-9} \text{ C}$$

$$|\vec{F}| = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^2}$$

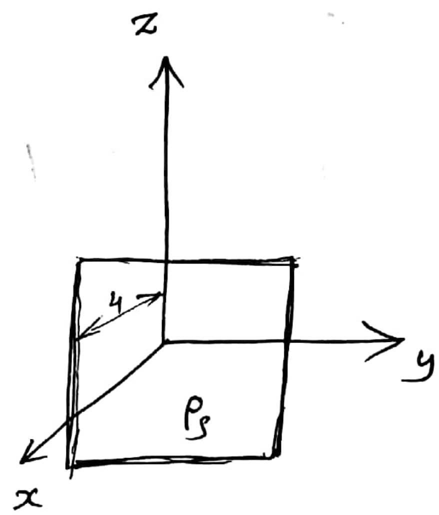
$$= \frac{(0.75\times10^{-9})^2}{4\pi \times 8.854\times10^{-12} \times (4\times10^{-2})^2}$$

$$\boxed{|\vec{F}| = 3.159 \times 10^{-6} \text{ N}}$$

Univ Question

A plane $x = 4$ Carry a surface charge density $10\ nC/m^2$ and a line $x = 0,\ z = 2$ carry a line charge density $10\ nC/m$. Calculate $\vec{E}$ at $(1, 1, -1)$ due to these two charges.

SOLUTION

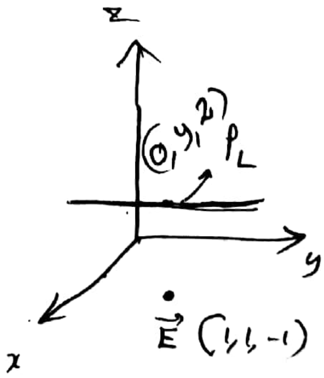


For surface charge density,

$$\vec{E}_1 = \frac{\rho_s}{2\epsilon_0}\,\hat{a}_x$$

$$\vec{E}_1 = \frac{10 \times 10^{-9}}{2 \times 8.854 \times 10^{-12}}\,\hat{a}_x$$

$$\boxed{\vec{E}_1 = 564\ \hat{a}_x\ \ V/m}$$



For line charge density

$$\vec{E}_2 = \frac{\rho_L}{2\pi\epsilon_0 R}\,\hat{a}_R$$

$$\vec{R} = (1, 1, -1) - (0, y, 2)$$

$$\vec{E}_2 = \frac{\rho_L}{2\pi\epsilon_0 R^3}\,\vec{R}$$

$$\vec{R} = \hat{a}_x - 3\hat{a}_z$$

$$|\vec{R}| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$\vec{E} = \frac{10 \times 10^{-9}}{2\pi \times 8.854 \times 10^{-12}\,(\sqrt{10})^2}\left(\hat{a}_x - 3\hat{a}_z\right)$$

~~$\vec{E}_2 = 63.55\ \hat{a}_x - 63.55\ \hat{a}_y$~~

~~$\vec{E} = \vec{E}_1 + \vec{E}_2$~~ $\quad \vec{E}_2 = 18.01\ \hat{a}_x - 54.03\ \hat{a}_z$

~~$\vec{E}_2 = 5.6843\ \hat{a}_x - 17.05\ \hat{a}_z$~~

$\vec{E} = \vec{E}_1 + \vec{E}_2$

~~$\vec{E} = 564\ \hat{a}_x + 5.6843\ \hat{a}_x - 17.05\ \hat{a}_z$~~

$$\boxed{\vec{E} = 582.01\ \hat{a}_x - 54.03\ \hat{a}_z}$$

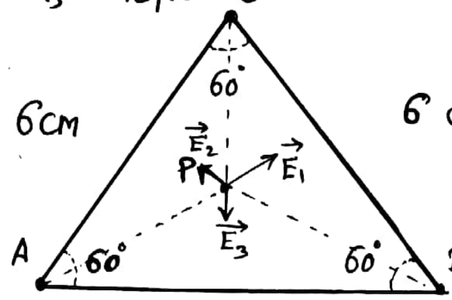
//

UNIV Q 1 :

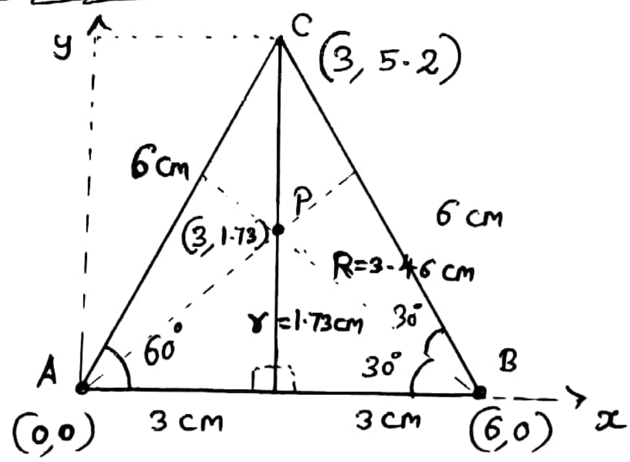
Charges of $10\mu C$, $20\mu C$ and $-15\mu C$ are placed at the corners of an equilateral triangle which has sides 6 cm long. The dielectric constant of the medium is 3. Find the electric field intensity at a point at the centre of the triangle equidistant from all the three charges.

SOLUTION

$\varepsilon_r = 3$



To find the Co-ordinates



$\sin 60° = \frac{y}{6}$

$y = 6 \sin 60°$

$y = 5.2$ cm

~~cos 60°~~

$\tan 30° = \frac{r}{3}$

$r = 3 \tan 30°$

$r = 1.73$ cm

$R = \sqrt{1.73^2 + 3^2}$

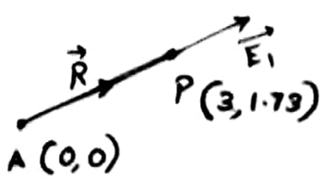
$R = 3.46$ cm

CASE 1

Electric field intensity at point 'P' due to $10\mu C$ at point 'A'

$$\vec{E}_1 = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_r R^3}\vec{R}$$

$$\vec{R} = 3\hat{a}_x + 1.73\hat{a}_y$$



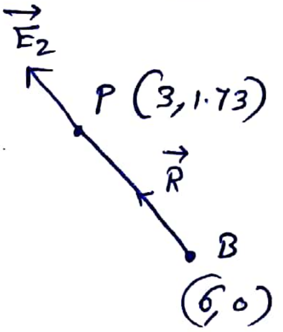
$$\vec{E}_1 = \frac{10\times10^{-6}}{4\pi\times8.854\times10^{-12}\times3\times(3.46\times10^{-2})^3}\left[3\hat{a}_x + 1.73\hat{a}_y\right]$$

$$\vec{E}_1 = 723.27\times10^{6}\,(3\hat{a}_x + 1.73\hat{a}_y)$$

$$\boxed{\vec{E}_1 = 2169.8\,\hat{a}_x + 1251.2\,\hat{a}_y}\times10^{6}$$

CASE 2

Electric field intensity at point 'P' due to $20\mu C$ at point 'B'



$$\vec{E}_2 = \frac{Q_2}{4\pi\varepsilon_0\varepsilon_r R^3}\vec{R}$$

$$\vec{R} = (3-6)\hat{a}_x + 1.73\hat{a}_y$$

$$\vec{R} = -3\hat{a}_x + 1.73\hat{a}_y$$

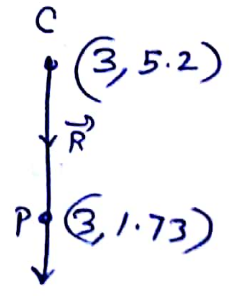
$$\vec{E}_2 = \frac{20\times10^{-6}}{4\pi\times8.854\times10^{-12}\times3\times(3.46\times10^{-2})^3}(-3\hat{a}_x + 1.73\hat{a}_y)$$

$$\vec{E}_2 = 1446.5\times10^{6}(-3\hat{a}_x + 1.73\hat{a}_y)$$

$$\boxed{\vec{E}_2 = -4339.6\times10^{6}\,\hat{a}_x + 2502.5\times10^{6}\,\hat{a}_y}$$

CESE 3

Electric field intensity at point 'P' due to $-15\mu C$ at point C



$$\vec{E}_3 = \frac{Q_3}{4\pi\varepsilon_0\varepsilon_r R^3}\vec{R}$$

$$\vec{R} = (3-3)\hat{a}_x + (5.2-1.73)\hat{a}_y$$

$$\vec{R} = -3.47\,\hat{a}_y$$

$$\vec{E}_3 = \frac{-15 \times 10^{-6}}{4\pi \times 8.854 \times 10^{-12} \times 3 \times (3.46 \times 10^{-2})^3} (-3.47\, \hat{a}_y)$$

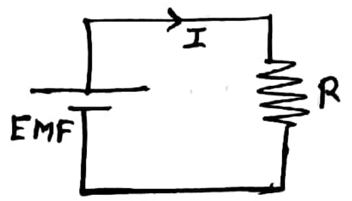
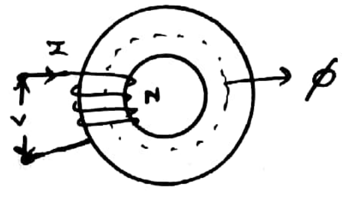
$$\vec{E}_3 = -1084.9 \times 10^{6} (-3.47\, \hat{a}_y)$$

$$\vec{E}_3 = 3764.6 \times 10^{6}\, \hat{a}_y$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$\vec{E} = (-2169.8\, \hat{a}_x + 7518.3\, \hat{a}_y) \times 10^{6}\ \text{V/m}$$

INTRODUCTION TO MAGNETIC CIRCUITS

ELECTRIC CIRCUIT	MAGNETIC CIRCUIT
	
EMF	MMF = NI
EMF ⇒ Electro Motive Force	MMF → Magneto Motive Force.
EMF = I R	MMF = ϕ S
I → Current R → Resistance	ϕ → Flux S → Reluctance
	Reluctance opposes the setting up of flux
$R = \frac{\rho l}{A}$	$S = \frac{l}{\mu A}$
$G = \frac{1}{R}$ G → Conductance.	$P = \frac{1}{S}$ P → Permeance.
	$H = \frac{NI}{l}$; $B = \frac{\phi}{A}$
	H → Magnetic field intensity

INDUCED EMF

Basically there are two types of induced EMF

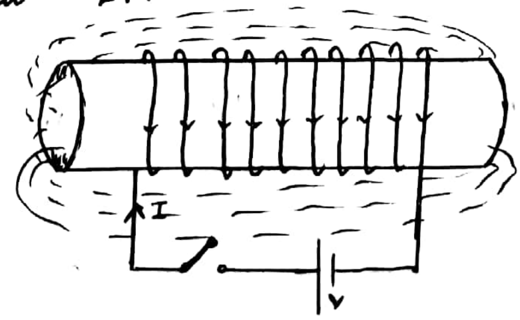
1) Statically induced EMF
2) Dynamically induced EMF

STATICALLY INDUCED EMF:

(i) Self induced EMF
(ii) Mutually induced EMF

SELF INDUCED EMF

When the current flowing through the coil is changed, the flux linking with the own winding changes and due to the change in linking flux with the coil an emf is induced and that EMF is known as self induced EMF



$\lambda \rightarrow$ flux linkage
N $\rightarrow$ Number of turns.
B $\rightarrow$ Flux density.

According to Faraday's law

$$e = -\frac{d\lambda}{dt} = -\frac{d(N\phi)}{dt} = -N\frac{d\phi}{dt}$$

$$e = -N\frac{d(BA)}{dt} = -N\frac{d(\mu H A)}{dt} \quad \left\{ \because B = \frac{\phi}{A},\ B = \mu H,\ H = \frac{NI}{l} \right.$$

$$e = -N\frac{d}{dt}\frac{\mu(NI)}{l}A$$

$$\left\{ L = \frac{\mu N^2 A}{l} \right\}$$

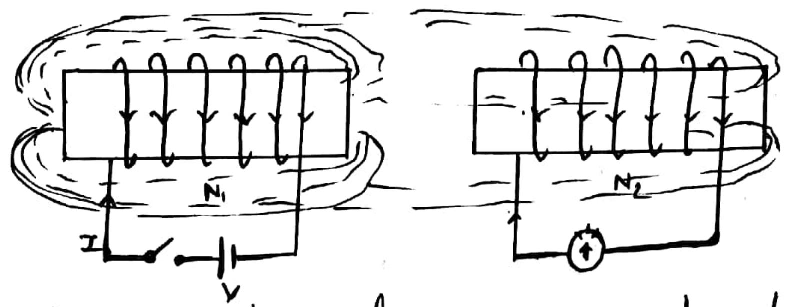
where,
$L \rightarrow$ Self inductance of the Coil.

$$e = -\frac{\mu N^2 A}{l}\left(\frac{dI}{dt}\right)$$

$$\boxed{e = -L\frac{dI}{dt}}$$

MUTUALLY INDUCED EMF

When two coils A and B placed together, then the flux created by one coil completely links with the other coil, and this phenomena is called Mutually induced EMF.



According to Faradays law, EMF induced in second Coil is given by

$$e_m = -N_2 \frac{d\phi_1}{dt}$$

$$e_m = -N_2 \frac{dB_1A}{dt} = -N_2 \frac{d\mu H_1 A}{dt}$$

$$e_m = -N_2 \frac{d\mu N_1 I_1}{dt \; l} A$$

$$e_m = \frac{-\mu N_1 N_2 A}{l} \left[\frac{dI_1}{dt}\right]$$

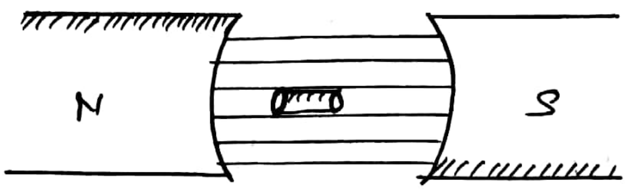
$$\boxed{e_m = -M \frac{dI_1}{dt}}$$

$$M = \frac{\mu N_1 N_2 A}{l}$$

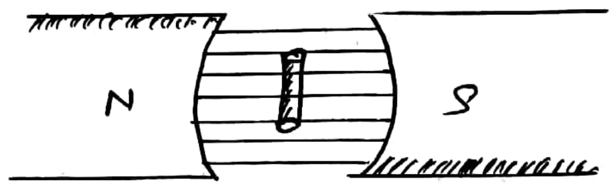
$M \rightarrow$ Coefficient of Mutual inductance.

DYNAMICALLY INDUCED EMF

An EMF is induced due to physical movement of coil with respect to flux (or) movement of magnet with respect to stationary coil is called dynamically induced EMF (or) Motional induced EMF.



Conductor parallel to Magnetic field line, EMF = 0



Conductor ⊥r to Magnetic field line, EMF → Maximum.

Let us consider the conductor moves through a small distance 'dx', in time 'dt'

According to Faraday's law, $e = \frac{d\phi}{dt}$

$$\left\{ \begin{array}{l} N = 1 \\ d\phi = B\,dA \\ dA = l\,dx \end{array} \right.$$

$$e = \frac{d(BA)}{dt}$$

$$e = \frac{B\,l\,dx}{dt} = Blv \text{ Volts}$$

For 360° rotation, $\boxed{e = Blv \sin\theta}$ volts

B → Flux density
l → Active length
v → velocity

DIRECTION

* Direction of induced EMF is given "FLEMING'S RIGHT HAND RULE"

FORCE AND TORQUE CALCULATIONS

* In a electric field, the force acting on a charged particle is given by

$$\boxed{F_E = QE}$$

Q → charge
E → Electric field.

* In a Magnetic field of flux density (B), a charged particle moving with velocity (v) experience a force which is given by

$$\boxed{F_M = Qv \times B}$$

* Force on a moving particle is due to Force on electric and Magnetic field is given by

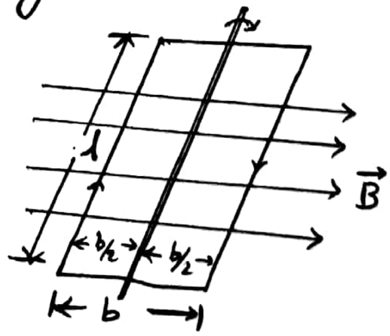
$$F = F_E + F_M = QE + Qv \times B$$

$$\boxed{F = Q(E + v \times B)}$$ Newton

MAGNETIC TORQUE

In general, Force acting on the loop is given by

$$F = BIl \sin\theta$$



The total torque on the loop

$$T = 2 \times \text{Torque on each side}$$

$$T = 2 \times \text{Force} \times \text{distace}$$

$$T = 2 \times BIl \sin\theta \times b/2$$

$$T = BI(lb)\sin\theta$$

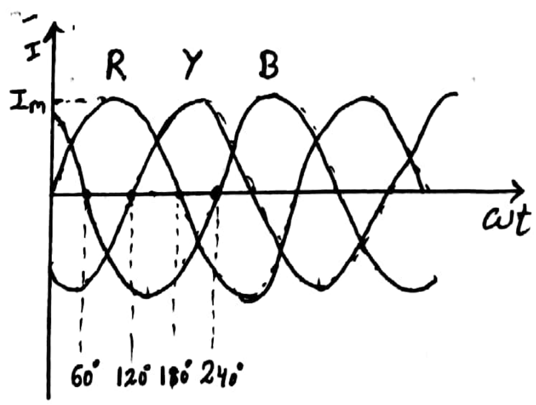
$l \times b = A$

$$\boxed{T = BIA \sin\theta}$$ N-m

MAGNETIC FIELD IN ROTATING MACHINE

"The magnetic field having constant amplitude but whose axis continuously rotates in a plane with a certain speed is called rotating magnetic field"

* When three phase supply is given to three phase winding machine (eg; 3ø Induction motor), the magnetic field strength keeps on changing in its 3ø winding.
* This change takes place at a speed of supply frequency (usually 50 Hz / 20 ms)
* The change in field strength looks like rotating magnetic field.

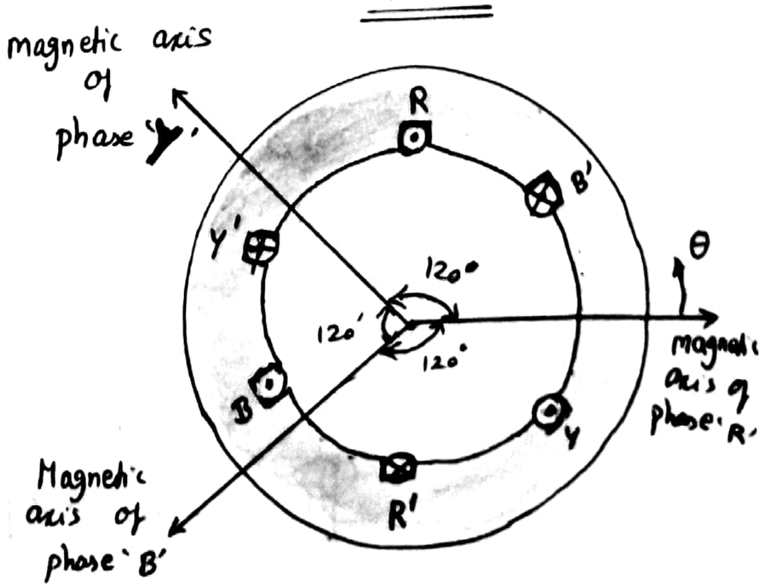


$I_R = I_m \cos \omega t$

$I_y = I_m \cos(\omega t - 120°)$

$I_b = I_m \cos(\omega t - 240°)$

RELATIVE LOCATION OF THE MAGNETIC AXES OF THREE PHASE



GENERATED VOLTAGE IN 'AC' MACHINE

According to Faradays law of induction.

$$e = -\frac{d\phi}{dt}$$

* Assume single turn coil
* '–' sign due to Lenz's law

For one complete revolution

$$d\phi = P\phi$$

$$dt = \frac{60}{N}$$

- $P \rightarrow$ No. of poles
- $\phi \rightarrow$ Total flux per pole

$$\therefore e = \frac{P\phi}{\frac{60}{N}} = \frac{NP\phi}{60}$$

WKT,

$$N = \frac{120f}{P}$$

$$\therefore e = \frac{120f}{P} \times \frac{P\phi}{60}$$

$$\boxed{e = 2f\phi} \longrightarrow \text{for single conductor}$$

$$e = 2f\phi Z \longrightarrow \text{for } Z \text{ conductor}$$

$$e = 4f\phi T \longrightarrow Z = 2T \rightarrow T \rightarrow \text{No. of turns}$$

$$e = 1.11 \; 4f\phi T$$

$$\{e = 4.44 f\phi T\} \text{ volt} \longrightarrow \{\text{RMS value} = \text{Form factor} \times \text{Average value}$$

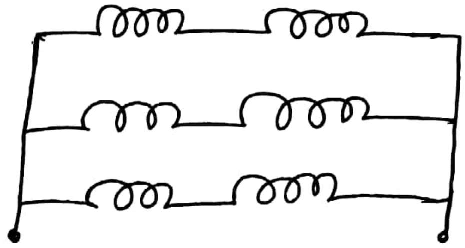
GENERATED VOLTAGE IN DC MACHINE

According to faradays law

$$e = \frac{d\phi}{dt}$$

$$e = \frac{P\phi}{\frac{60}{N}} = \frac{NP\phi}{60}$$

Emf induced in z conductor



$$e = \frac{NP\phi}{60} \cdot z$$

here

No. of parallel path, $A = 3$

No. of conducto, $z = 6$

Conductor / Parallel Path

$z/A = 2$

Emf induced per parallel path

$$e = \frac{NP\phi}{60} \frac{z}{A}$$

$$\therefore \left\{ e = \frac{\phi z N}{60} \cdot \frac{P}{A} \right\} \text{Volt}$$

for lap winding	for wave winding
$A = P$	$A = 2$

Convert points P (1, 3, 5), T (0, -4, 3) & S (-3, -4, -10) from Cartesian to Cylindrical and Spherical Co-ordinates

SOLUTION

P (1, 3, 5)

Cylindrical (ρ, ϕ, z)

$\rho = \sqrt{x^2+y^2} = \sqrt{1^2+3^2} = 3.162$

$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{3}{1} = 71.56°$

$z = z = 5 = 5$

$P[3.162, 71.56°, 5]$

Spherical (r, θ, ϕ)

$r = \sqrt{x^2+y^2+z^2} = 5.916$

$\theta = \tan^{-1}\frac{\rho}{z}$ (or) $\theta = \cos^{-1}\frac{z}{r}$

$\theta = \tan^{-1}\frac{3.162}{5}$; $\theta = \cos^{-1}\frac{5}{5.916}$

$\theta = 32.31°$; $\theta = 32.31°$

$\phi = 71.56°$

$P(5.916, 32.31°, 71.56°)$

(2) T (4, 270°, 3)

Express the following points in cylindrical and spherical Co-ordinates.

P(1, -4, 3) ; Q (3, 0, 5) (c) R(-2, 6, 0)

~~[illegible]~~, Following points to cartesian co-ordinates.

$P_1(2, 30°, 5)$ $P_2(1, 90°, -3)$ $P_3(10, \pi/4, \pi/3)$; $P_4(4, 30°, 60°)$

(1) a) (4.123, 284.04°, -3) (5.099, 126.04°, 284.40°)

b) (3, 0°, 5) (5.831, 30.96°, 0°)

c) (6.325, 108.4°, 0) (6.325, 90°, 108.4°)

---

(2) P_1 (1.732, 1, 5) P_3 (3.535, 6.124, 7.0711)

P_2 (0, 1, -3) P_4 (1, 1.7321, 3.464)